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CP violation in polarized $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ decays

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Abstract. We study the decay rate and the CP violating asymmetry of the exclusive $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ decays in the case where one of the final leptons is polarized. We calculate the contributions coming from the individual polarization states in order to identify a so-called wrong sign decay, which is a decay with a given polarization, whose width and CP asymmetry are smaller as compared to the unpolarized one. The results are presented for electrons and tau leptons. We observe that in particular decay channels, one can identify a wrong sign decay which is more sensitive to new physics beyond the standard model.

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1 Introduction

The rare B decays, which are induced at quark level by flavor changing neutral currents (FCNC), have received a lot of attention, since they are very promising for investigating the standard model (SM) and searching for the new physics beyond it. Among these B decays, the rare semileptonic ones have played a central role for a long time, since they offer the most direct methods to determine the weak mixing angles and Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. These decays can also be very useful to test the various new physics scenarios like the two Higgs doublet models (2HDM), minimal supersymmetric standard model (MSSM) [1], etc.

On the experimental side, there is an impressive effort to search for B decays, in B-factories such as Belle, BaBar and LHC-B. The CLEO Collaboration reports for the branching ratios (BR) of the $B^0 \to \pi^-\ell^+\nu$ and $B^0 \to \rho^-\ell^+\nu$ decays [2]

$$\begin{aligned} \mathrm{BR}(B^0 \to \pi^-\ell^+\nu) &= (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}\,, \\ \mathrm{BR}(B^0 \to \rho^-\ell^+\nu) &= (2.57 \pm 0.29^{+0.33}_{-0.46} \pm 0.41) \times 10^{-4}. \end{aligned} \tag{1}$$

From these results, the value of the CKM matrix element $|V_{ub}| = 3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55$ has been determined [2]. Recently, the BR of the inclusive $B \to X_s\ell^+\ell^-$ decay has been also reported by the Belle Collaboration [3];

$$\mathrm{BR}(B \to X_s\ell^+\ell^-) = (6.1 \pm 1.4^{+1.4}_{-1.1}) \times 10^{-6}\,, \tag{2}$$

which is very close to the value predicted by the SM [4]. The experimental result from the BaBar Collaboration for this BR is [5]

$$\mathrm{BR}(B \to X_s\ell^+\ell^-) = (6.3 \pm 1.6^{+1.8}_{-1.5}) \times 10^{-6}\,. \tag{3}$$

From the theoretical point of view exclusive channels are harder to evaluate than inclusive channels, because exclusive channels require additional knowledge of the form factors, which are used to incorporate hadronic effects. However, the exclusive channels are easier to measure. The decay channels that are induced by the $b \to d\ell^+\ell^-$ decay at the quark level are promising for searching CP violation. For the B decays that are induced by the decay $b \to s\ell^+\ell^-$, the terms which describe virtual effects as $t\bar{t}$, $c\bar{c}$ and $u\bar{u}$ loops are in the matrix element proportional to $V_{tb}V_{ts}^*$, $V_{ub}V_{us}^*$ and $V_{cb}V_{cs}^*$ respectively. Because of the unitary property of the CKM matrix and because of the fact that $V_{ub}V_{us}^*$ is small compared to the other CKM factors, the CP violation is strongly suppressed in these decays [6,7]. Although the BR of the B decays induced by $b \to d\ell^+\ell^-$ are smaller, the CKM factors $V_{tb}V_{td}^*$, $V_{ub}V_{ud}^*$ and $V_{cb}V_{cd}^*$ are all of the same order. Therefore CP violation is much more considerable in these decays [8]. In this context, the exclusive $B_d \to (\pi, \rho, \eta, \eta')\,\ell^+\ell^-$, and $B_d \to \gamma\,\ell^+\ell^-$ decays have been extensively studied in the SM [9–11] and beyond [12–16].

In [17], it has been observed that the unpolarized CP asymmetry and decay width for the inclusive $b \to d\ell^+\ell^-$ decay are comparable to the CP asymmetry and decay width when one of the leptons is in a specific polarization state. The CP asymmetry as well as the decay rate in the case of the other polarization state turn out to be smaller as compared to the unpolarized spectrum and in [17] this is defined as the *wrong sign* polarized state. Along this

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line, in [18], a similar analysis of the CP asymmetries in $b \to d\ell^+\ell^-$ decays has been performed in a model independent way and it was reported that polarized asymmetries are very sensitive to various new Wilson coefficients. In this paper, motivated by the works in [17,18], we make a similar analysis of the exclusive $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ channels and calculate the contributions coming from the individual polarization states in order to identify a wrong sign decay. This feature can provide measurements involving a new physics search.

Our paper is organized as follows. In Sect. 2 we present the effective Hamiltonian and derive the expressions for the unpolarized and the polarized differential decay rates of $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$. The CP violating asymmetries for these decays in the unpolarized as well as in the polarized case are calculated in Sect. 3. The numerical results and the discussions are presented in Sect. 4, which is followed by a conclusion section.

2 Exclusive $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ decays

2.1 Effective Hamiltonian

The leading order QCD corrected effective Hamiltonian, which is induced by the corresponding quark level process $b \to d\,\ell^+\ell^-$, is given by [19–22]

$$\mathcal{H}_{\rm eff} = \frac{4G_{\rm F}\,\alpha}{\sqrt{2}}\,V_{tb}V_{td}^* \times \Bigg\{\sum_{i=1}^{10} C_i(\mu)\,O_i(\mu) -\lambda_u\{C_1(\mu)[O_1^u(\mu)-O_1(\mu)] + C_2(\mu)[O_2^u(\mu)-O_2(\mu)]\}\Bigg\}, \quad (4)$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}, \quad (5)$$

using the unitarity of the CKM matrix i.e., $V_{tb}V_{td}^* + V_{ub}V_{ud}^* = -V_{cb}V_{cd}^*$. The explicit forms of the operators O_i can be found in [19,22]. In (4), $C_i(\mu)$ are the Wilson coefficients calculated at a renormalization point μ and their evolution from the higher scale $\mu = m_W$ down to the low-energy scale $\mu = m_b$ is described by the renormalization group equation. For $C_7^{\rm eff}$ this calculation is performed up to next-to-next-to-leading logarithmic (NNLL) order in [23–25], while $C_9^{\rm eff}$ and C_{10} were calculated in [26]. In the context of the SM NNLL QCD corrections to the BR [26–30] and the forward–backward asymmetry [31–34] in $B \to X_s\ell^+\ell^-$ are also available. For a recent review see e.g. [35]. The corresponding NNLL results for $B \to X_d\ell^+\ell^-$ are given in [36].

The term that is the source of the CP violation can be parameterized as follows:

$$C_9^{\rm eff} = \xi_1 + \lambda_u\xi_2, \quad (6)$$

where

$$\begin{aligned}\xi_1 = {} & C_9 + g(\hat m_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ & -\frac{1}{2}g(\hat m_d, s)(C_3 + C_4) \\ & -\frac{1}{2}g(\hat m_b, s)(4C_3 + 4C_4 + 3C_5 + C_6) \\ & +\frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned} \quad (7)$$

and

$$\xi_2 = [g(\hat m_c, s) - g(\hat m_u, s)](3C_1 + C_2). \quad (8)$$

In (7) and (8), $s = q^2/m_B^2$, where q is the momentum transfer and $\hat m_q = m_q/m_b$. The functions $g(\hat m_q, s)$ arise from one loop contributions of the four-quark operators O_1–O_6 and are given by

$$\begin{aligned} g(\hat m_q, s) = {} & -\frac{8}{9}\ln\hat m_q + \frac{8}{27} + \frac{4}{9}y \\ & -\frac{2}{9}(2+y)|1-y|^{1/2} \\ & \times \begin{cases} \left(\ln\left|\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\right| - {\rm i}\pi\right), & \text{for } y \equiv \frac{4\hat m_q^2}{s} < 1 \\ 2\arctan\frac{1}{\sqrt{y-1}}, & \text{for } y \equiv \frac{4\hat m_q^2}{s} > 1\,. \end{cases} \end{aligned} \quad (9)$$

The $C_9^{\rm eff}$ term receives also contributions from long-distance effects. The $c\bar c$ resonance can be parameterized by means of a Breit–Wigner shape [37]. It is incorporated in the $C_9^{\rm eff}$ term by the following replacement:

$$g(\hat m_c, s) \to g(\hat m_c, s) - \frac{3\pi}{\alpha^2}\kappa \sum_{V=J/\psi,\psi',\ldots} \frac{m_V {\rm BR}(V \to \ell^+\ell^-)\Gamma_{\rm total}^V}{s m_B^2 - m_V^2 + {\rm i} m_V \Gamma_{\rm total}^V}. \quad (10)$$

To reproduce the correct experimental BR for ${\rm BR}(B \to J/\psi X \to X\ell\bar\ell) = {\rm BR}(B \to J/\psi X){\rm BR}(J/\psi \to X\ell\bar\ell)$, the factor κ is taken to be 2.3 [37].

Neglecting the mass of the d quark, the effective short-distance Hamiltonian for the $b \to d\ell^+\ell^-$ decay in (4) leads to the QCD corrected matrix element:

$$\begin{aligned}\mathcal{M} = {} & \frac{G_{\rm F}\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^* \\ & \times\Bigg\{C_9^{\rm eff}(m_b)\;\bar d\gamma_\mu(1-\gamma_5)b\,\bar\ell\gamma^\mu\ell \\ & + C_{10}(m_b)\;\bar d\gamma_\mu(1-\gamma_5)b\,\bar\ell\gamma^\mu\gamma_5\ell \\ & -2C_7^{\rm eff}(m_b)\;\frac{m_b}{q^2}\bar d{\rm i}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar\ell\gamma^\mu\ell\Bigg\}. \end{aligned} \quad (11)$$

2.2 The exclusive $B \to \pi \ell^+ \ell^-$ decay

In this section we present the expressions for the differential decay rate of $B \to \pi \ell^+ \ell^-$ decay with both unpolarized and polarized leptons. For this purpose, we need the following matrix elements, which are written in terms of the form factors:

$$\begin{aligned} &\langle \pi(p_\pi) | \bar{d} \gamma_\mu (1-\gamma^5) b | B(p_B) \rangle \\ &\quad = f^+(q^2)(p_B+p_\pi)_\mu + f^-(q^2) q_\mu, \qquad (12) \\ &\langle \pi(p_\pi) | \bar{d} \mathrm{i} \sigma_{\mu\nu} q^\nu (1+\gamma^5) b | B(p_B) \rangle \\ &\quad = [q^2 (p_B+p_\pi)_\mu - q_\mu (m_B^2 - m_\pi^2)] f_v(q^2). \qquad (13) \end{aligned}$$

Here, p_π and p_B are the 4-momenta of the π and the B meson, respectively. Also f^+, f^- and $f_v = -\frac{f_T}{m_B+m_\pi}$ represent the relevant form factors.

From (11), and using the matrix elements in (12) and (13), we obtain the amplitude governing the $B \to \pi \ell^+ \ell^-$ decay:

$$\begin{aligned} \mathcal{M}^{B\to\pi} = & \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \qquad (14) \\ & \times \{ (2A p_\pi^\mu + B q^\mu) \bar{\ell} \gamma_\mu \ell + (2G p_\pi^\mu + D q^\mu) \bar{\ell} \gamma_\mu \gamma^5 \ell \}, \end{aligned}$$

where

$$\begin{aligned} A &= C_9^{\mathrm{eff}} f^+ - 2 m_B C_7^{\mathrm{eff}} f_v, \\ B &= C_9^{\mathrm{eff}} (f^+ + f^-) + 2 \frac{m_B}{q^2} C_7^{\mathrm{eff}} f_v (m_B^2 - m_\pi^2 - q^2), \\ G &= C_{10} f^+, \qquad (15) \\ D &= C_{10}(f^+ + f^-). \end{aligned}$$

Using the matrix element in (14), performing summation over final lepton polarizations and integrating over angle variables, the unpolarized differential decay width is obtained as

$$\left(\frac{\mathrm{d}\Gamma^\pi}{\mathrm{d}s} \right)_0 = \frac{G_F^2 \alpha^2}{2^{10} \pi^5} |V_{tb} V_{td}{}^*|^2 m_B^3 \, v \, \sqrt{\lambda_\pi} \, \Delta_\pi, \qquad (16)$$

where

$$\begin{aligned} \Delta_\pi = & \frac{1}{3} \, m_B^2 \, \lambda_\pi (3 - v^2)(|A|^2 + |G|^2) \\ & + 16 \, m_\ell^2 \, r_\pi \, |G|^2 + 4 \, m_\ell^2 \, s \, |D|^2 \\ & + 8 \, m_\ell^2 \, (1 - r_\pi - s) \mathrm{Re}[G D^*], \qquad (17) \end{aligned}$$

with $r_\pi = m_\pi^2/m_B^2$, $\lambda_\pi = r_\pi^2 + (s-1)^2 - 2 r_\pi (s+1)$, $v = \sqrt{1 - \frac{4t^2}{s}}$ and $t = m_\ell/m_B$.

In order to calculate the polarized decay spectrum, we need the final lepton polarizations. For this, one defines orthogonal unit vectors $\boldsymbol{e}_\mathrm{L}$, $\boldsymbol{e}_\mathrm{T}$ and $\boldsymbol{e}_\mathrm{N}$ such that in the rest frame of ℓ^- lepton they are written as

$$\begin{aligned} S_\mathrm{L}^\mu &\equiv (0, \boldsymbol{e}_\mathrm{L}) = \left(0, \frac{\boldsymbol{p}_1}{|\boldsymbol{p}_1|} \right), \\ S_\mathrm{N}^\mu &\equiv (0, \boldsymbol{e}_\mathrm{N}) = \left(0, \frac{\boldsymbol{k} \times \boldsymbol{p}_1}{|\boldsymbol{k} \times \boldsymbol{p}_1|} \right), \\ S_\mathrm{T}^\mu &\equiv (0, \boldsymbol{e}_\mathrm{T}) = \left(0, \boldsymbol{e}_\mathrm{N}^- \times \boldsymbol{e}_\mathrm{L}^- \right). \qquad (18) \end{aligned}$$

Here, $\boldsymbol{p}_1$ is the 3-vector of the ℓ^- lepton and $\boldsymbol{k}$ is the 3-vector of the final meson. The longitudinal unit vector S_L is boosted to the CM frame of $\ell^+\ell^-$ by the Lorentz transformation:

$$S_{\mathrm{L,CM}}^\mu = \left(\frac{|\boldsymbol{p}_1|}{m_\ell}, \frac{E_\ell \, \boldsymbol{p}_1}{m_\ell |\boldsymbol{p}_1|} \right), \qquad (19)$$

while S_T and S_N are not changed by the boost. The differential decay rate of the $B \to \pi \ell^+ \ell^-$ decay, for any spin direction $\boldsymbol{n}$ of ℓ^-, can be written in the following form:

$$\frac{\mathrm{d}\Gamma^\pi(s, \boldsymbol{n})}{\mathrm{d}s} = \frac{1}{2} \left(\frac{\mathrm{d}\Gamma^\pi}{\mathrm{d}s} \right)_0 [1 + P_i^\pi \boldsymbol{e}_i \cdot \boldsymbol{n}], \qquad (20)$$

where a sum over $i = \mathrm{L, T, N}$ is implied. The polarization components P_i^π in (20) are defined as

$$P_i^\pi(s) = \frac{\mathrm{d}\Gamma^\pi(\boldsymbol{n} = \boldsymbol{e}_i)/\mathrm{d}s - \mathrm{d}\Gamma^\pi(\boldsymbol{n} = -\boldsymbol{e}_i)/\mathrm{d}s}{\mathrm{d}\Gamma^\pi(\boldsymbol{n} = \boldsymbol{e}_i)/\mathrm{d}s + \mathrm{d}\Gamma^\pi(\boldsymbol{n} = -\boldsymbol{e}_i)/\mathrm{d}s}. \qquad (21)$$

The resulting expressions for the polarization asymmetries are obtained as

$$\begin{aligned} P_\mathrm{L}^\pi &= \frac{4 m_B^2}{3 \Delta_\pi} \, v \, \lambda_\pi \, \mathrm{Re}[A G^*], \\ P_\mathrm{T}^\pi &= \frac{m_B^2}{\sqrt{s} \Delta_\pi} \, \sqrt{\lambda_\pi} \, \pi \, t \qquad (22) \\ & \quad \times (\mathrm{Re}[A D^*] s + \mathrm{Re}[A G^*](1 - r_\pi - s)), \\ P_\mathrm{N}^\pi &= 0. \end{aligned}$$

Our results for P_L^π and P_T^π agree with the ones given in [38]. As can be seen from the explicit expressions of P_i^π, the polarization P_T^π is proportional to m_ℓ and therefore can be significant for τ lepton only.

2.3 The exclusive $B \to \rho \ell^+ \ell^-$ decay

In this section we present the expressions for the differential decay rate for $B \to \rho \ell^+ \ell^-$ decay with both unpolarized and polarized leptons. For this, we need the following matrix elements:

$$\begin{aligned} &\langle \rho(p_\rho, \varepsilon) | \bar{d} \gamma_\mu (1 - \gamma_5) b | B(p_B) \rangle \\ &\quad = -\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda p_B^\sigma \frac{2V(q^2)}{m_B + m_\rho} - \mathrm{i} \varepsilon_\mu^* (m_B + m_\rho) A_1(q^2) \\ &\qquad + \mathrm{i} (p_B + p_\rho)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_\rho} \\ &\qquad + \mathrm{i} q_\mu (\varepsilon^* q) \frac{2 m_\rho}{q^2} [A_3(q^2) - A_0(q^2)], \qquad (23) \\ &\langle \rho(p_\rho, \varepsilon) | \bar{d} \mathrm{i} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle \end{aligned}$$

$$= 4\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_\rho^\lambda q^\sigma T_1(q^2) \quad (24)$$
$$+2\mathrm{i}[\varepsilon^*_\mu(m_B^2 - m_\rho^2) - (p_B + p_\rho)_\mu(\varepsilon^* q)]T_2(q^2)$$
$$+2\mathrm{i}(\varepsilon^* q)\left(q_\mu - (p_B+p_\rho)_\mu \frac{q^2}{m_B^2 - m_\rho^2}\right)T_3(q^2),$$
$$\langle\rho(p_\rho,\varepsilon)|\bar{d}(1+\gamma_5)b|B(p_B)\rangle = \frac{-1}{m_b}2\mathrm{i}m_\rho(\varepsilon^* q)A_0(q^2)\,, \quad (25)$$

where p_ρ and ε denote the 4-momentum and polarization vectors of the ρ meson, respectively.

From (23)–(25), we get the following expression for the matrix element of the $B \to \rho\ell^+\ell^-$ decay:

$$\mathcal{M}^{B\to\rho} = \frac{G_\mathrm{F}\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^*$$
$$\times\{\bar{\ell}\gamma_\mu(1-\gamma_5)\ell[2A\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_\rho^\lambda p_B^\sigma + \mathrm{i}B\varepsilon^*_\mu$$
$$-\mathrm{i}C(p_B+p_\rho)_\mu(\varepsilon^* q) - \mathrm{i}D(\varepsilon^* q)q_\mu]$$
$$+\bar{\ell}\gamma_\mu(1+\gamma_5)\ell[2E\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_\rho^\lambda p_B^\sigma + \mathrm{i}F\varepsilon^*_\mu$$
$$-\mathrm{i}G(\varepsilon^* q)(p_B+p_\rho) - \mathrm{i}H(\varepsilon^* q)q_\mu]\}\,, \quad (26)$$

where

$$A = (C_9^\mathrm{eff} - C_{10})\frac{V}{m_B+m_\rho} + 4\frac{m_b}{q^2}C_7^\mathrm{eff}T_1,$$
$$B = (m_B+m_\rho)$$
$$\times\left((C_9^\mathrm{eff} - C_{10})A_1 + \frac{4m_b}{q^2}(m_B^2-m_\rho^2)C_7^\mathrm{eff}T_2\right),$$
$$C = (C_9^\mathrm{eff} - C_{10})\frac{A_2}{m_B+m_\rho}$$
$$+4\frac{m_b}{q^2}C_7^\mathrm{eff}\left(T_2 + \frac{q^2}{m_B^2-m_\rho^2}T_3\right),$$
$$D = 2(C_9^\mathrm{eff} - C_{10})\frac{m_\rho}{q^2}(A_3 - A_0) - 4C_7^\mathrm{eff}\frac{m_b}{q^2}T_3,$$
$$E = A(C_{10}\to -C_{10}), \quad (27)$$
$$F = B(C_{10}\to -C_{10}),$$
$$G = C(C_{10}\to -C_{10}),$$
$$H = D(C_{10}\to -C_{10}).$$

Here A_0, A_1, A_2, A_3, V, T_1, T_2 and T_3 are the relevant form factors.

Using the matrix element in (26), we find the unpolarized differential decay width as

$$\left(\frac{\mathrm{d}\Gamma^\rho}{\mathrm{d}s}\right)_0 = \frac{\alpha^2 G_\mathrm{F}^2 m_B}{2^{12}\pi^5}|V_{tb}V_{td}^*|^2\, v\,\sqrt{\lambda_\rho}\,\Delta_\rho\,, \quad (28)$$

where

$$\Delta_\rho = \frac{8}{3}m_B^4\lambda_\rho$$
$$\times\left[(m_B^2 s - m_\ell^2)\left(|A|^2+|E|^2\right) + 6m_\ell^2\,\mathrm{Re}(AE^*)\right]$$
$$+24m_\ell^2\,\mathrm{Re}(BF^*) + \frac{1}{r_\rho}m_B^4 m_\ell^2 s\lambda_\rho\,|D-H|^2$$
$$+\frac{2}{r_\rho}m_B^2 m_\ell^2\lambda_\rho$$
$$\times(\mathrm{Re}[B(-D^*+G^*+H^*)]$$
$$+\mathrm{Re}[F(C^*+D^*-H^*)])$$
$$+\frac{1}{2}m_\ell\,\mathrm{Re}[(C-G)(D^*-H^*)]$$
$$-\frac{2}{r_\rho}m_B^4 m_\ell^2\lambda_\rho(2+2r_\rho-s)\,\mathrm{Re}(CG^*))$$
$$-\frac{2}{3r_\rho s}m_B^2\lambda_\rho\,\left[m_\ell^2(2-2r_\rho+s) + m_B^2 s(1-r_\rho-s)\right]$$
$$\times[\mathrm{Re}(BC^*) + \mathrm{Re}(FG^*)]$$
$$+\frac{1}{3r_\rho s}\,\left[2m_\ell^2(\lambda_\rho - 6r_\rho s) + m_B^2 s(\lambda_\rho + 12 r_\rho s)\right]$$
$$\times\left(|B|^2+|F|^2\right)$$
$$+\frac{1}{3r_\rho s}m_B^4\lambda_\rho$$
$$\times\left(m_B^2 s\lambda_\rho + m_\ell^2[2\lambda_\rho + 3s(2+2r_\rho-s)]\right)$$
$$\times\left(|C|^2+|G|^2\right)\,, \quad (29)$$

where $\lambda_\rho = r_\rho^2 + (s-1)^2 - 2r_\rho(s+1)$ and $r_\rho = m_\rho^2/m_B^2$.

The polarization components are obtained in the same way as in the previous section. The differential decay rate of the $B \to \rho\ell^+\ell^-$ decay, for any spin direction $\boldsymbol{n}$ of ℓ^-, can be written in the following form:

$$\frac{\mathrm{d}\Gamma^\rho(s,\boldsymbol{n})}{\mathrm{d}s} = \frac{1}{2}\left(\frac{\mathrm{d}\Gamma^\rho}{\mathrm{d}s}\right)_0\left[1 + P_i^\rho\boldsymbol{e}_i\cdot\boldsymbol{n}\right]\,, \quad (30)$$

where a sum over $i = \mathrm{L,T,N}$ is implied. The resulting expressions for the polarization asymmetries are obtained as

$$P_\mathrm{L}^\rho = \frac{-1}{3r_\rho\Delta_\rho}m_B^2 v$$
$$\times\left(8m_B^4 s r_\rho\lambda_\rho(|E|^2-|A|^2)\right.$$
$$-(12r_\rho s+\lambda_\rho)(|B|^2-|F|^2) + m_B^4\lambda_\rho^2(|G|^2-|C|^2)$$
$$\left.-2m_B^2\lambda_\rho(-1+r_\rho+s)\mathrm{Re}[CB^*-FG^*]\right)\,,$$
$$P_\mathrm{T}^\rho = \frac{-1}{4r_\rho\sqrt{s}\Delta_\rho}m_B m_\ell\pi\sqrt{\lambda_\rho}$$
$$\times\left(m_B^4\lambda_\rho(r_\rho-1)(|C|^2-|G|^2)\right)$$
$$+m_B^2 s(1+3r_\rho-s)\mathrm{Re}[CF^*-BG^*]$$
$$+8r_\rho s m_B^2\mathrm{Re}[(A+E)(B^*+F^*)]$$
$$+m_B^2$$

$$
\begin{aligned}
&\times(\lambda_\rho + (-1+r_\rho+s)(r_\rho-1))\mathrm{Re}[BC^* - FG^*] \\
&+(-1+r_\rho+s) \\
&\quad\times(|B|^2 - |F|^2 + s m_B^2 \mathrm{Re}[(B+F)(H^*-D^*)]) \\
&+m_B^4 s\lambda_\rho \mathrm{Re}[(C+G)(H^*-D^*)]\big) \ , \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{N}}^\rho = &\frac{1}{4 r_\rho \Delta_\rho} m_B^3 m_\ell \pi v \sqrt{s\lambda_\rho} \\
&\times (8\mathrm{Im}[EB^* + FA^*] \\
&-(1-r_\rho-s)\mathrm{Im}[(B-F)(D^*-H^*)] \\
&-(1+3r_\rho-s)\mathrm{Im}[(B-F)(C^*-G^*)] \\
&+m_B^2\lambda_\rho \mathrm{Im}\left[(C-G)(D^*-H^*)\right]\big) \ .
\end{aligned}
$$

Our results for P_{L}^ρ, P_{N}^ρ and P_{T}^ρ agree with those given in [39] for the SM case. As can be seen from the explicit expressions of P_i^ρ, they involve various quadratic combinations of the Wilson coefficients and hence they are quite sensitive to the new physics. The polarizations P_{N}^ρ and P_{T}^ρ are again proportional to m_ℓ as in the $B \to \pi \ell^+\ell^-$ decay and therefore can be significant for the τ lepton only.

3 CP violation

3.1 CP violating asymmetry in $B \to \pi \ell^+\ell^-$ decay

In $B \to \pi \ell^+\ell^-$ decay with unpolarized final leptons, CP violating differential decay width asymmetry is defined as

$$
A_{CP}^\pi(s) = \frac{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0}{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0} = \frac{\Delta_\pi - \bar{\Delta}_\pi}{\Delta_\pi + \bar{\Delta}_\pi} \ , \quad (32)
$$

where

$$
\frac{\mathrm{d}\Gamma^\pi}{\mathrm{d}s} = \frac{\mathrm{d}\Gamma(B \to \pi e^+ e^-)}{\mathrm{d}s}, \ \frac{\mathrm{d}\bar{\Gamma}^\pi}{\mathrm{d}s} = \frac{\mathrm{d}\Gamma(\bar{B} \to \bar{\pi} e^+ e^-)}{\mathrm{d}s} \ .
$$

In the SM, the Wilson coefficient C_9^{eff} is the only one that contributes to A_{CP} above since it has an imaginary component as well as a real one, which can be parameterized as in (6). Therefore, $(\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0$ and $\bar{\Delta}_\pi$ in (32) can be obtained from $(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0$ and Δ_π by making the replacement

$$
\left(\frac{\mathrm{d}\bar{\Gamma}^\pi}{\mathrm{d}s}\right)_0 = \left(\frac{\mathrm{d}\Gamma^\pi}{\mathrm{d}s}\right)_0 |_{\lambda_u \to \lambda_u^*} \quad , \quad \bar{\Delta}_\pi = \Delta_\pi \,|_{\lambda_u \to \lambda_u^*} \ . \quad (33)
$$

Using (17), (32) and (33), the CP violating asymmetry is obtained as

$$
A_{CP}(s) = \frac{-2\mathrm{Im}[\lambda_u]\Sigma_\pi(s)}{\Delta_\pi + 2\mathrm{Im}[\lambda_u]\Sigma_\pi(s)} \ ,
$$

where

$$
\begin{aligned}
\Sigma_\pi(s) = &\frac{1}{3} m_B^2 \lambda_\pi (3 - v^2) \qquad (34) \\
&\times\left({f^+}^2 \mathrm{Im}[\xi_1^*\xi_2] - 2m_B\ f_v f^+\ \mathrm{Im}[\xi_2 {C_7^{\mathrm{eff}}}^*]\right).
\end{aligned}
$$

When one of the leptons is polarized in $B \to \pi \ell^+\ell^-$ decay, the CP violating asymmetry can be defined as follows:

$$
A_{CP}^\pi(s, \boldsymbol{n}) = \frac{\mathrm{d}\Gamma^\pi(s,\boldsymbol{n})/\mathrm{d}s - \mathrm{d}\bar{\Gamma}^\pi(s, \bar{\boldsymbol{n}} = -\boldsymbol{n})/\mathrm{d}s}{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 + (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0} \ , \quad (35)
$$

where

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma^\pi(s,\boldsymbol{n})}{\mathrm{d}s} &= \frac{\mathrm{d}\Gamma^\pi(B \to \pi e^+ e^-(\boldsymbol{n}))}{\mathrm{d}s} \ , \\
\frac{\mathrm{d}\bar{\Gamma}^\pi(s,\bar{\boldsymbol{n}})}{\mathrm{d}s} &= \frac{B \to \pi e^+(\bar{\boldsymbol{n}}) e^-)}{\mathrm{d}s} \ .
\end{aligned}
$$

Here, $\bar{\boldsymbol{n}}$ is the spin direction of the ℓ^+ in the $\bar{B} \to \bar{\pi}\ell^+\ell^-$ decay. From the expression for the polarized differential decay width for the $B \to \pi \ell^+\ell^-$ decay given by (20), the width for the corresponding CP conjugated process reads

$$
\frac{\mathrm{d}\bar{\Gamma}^\pi(s,\bar{\boldsymbol{n}})}{\mathrm{d}s} = \frac{1}{2}\left(\frac{\mathrm{d}\bar{\Gamma}^\pi}{\mathrm{d}s}\right)_0 \left[1 + \bar{P}_i^\pi \bar{\boldsymbol{e}}_i \cdot \bar{\boldsymbol{n}}\right] \ . \quad (36)
$$

Since in the CP conserving case $\bar{P}_i^\pi = -P_i^\pi$, in the general case with the choice $\bar{\boldsymbol{e}}_i = \boldsymbol{e}_i$, $\bar{P}_i^\pi$ can be constructed by the replacement

$$
\bar{P}_i^\pi = -P_i^\pi \,|_{\lambda_u \to \lambda_u^*} \ . \quad (37)
$$

Inserting (20) and (36) into (35), and setting $\bar{\boldsymbol{n}} = \boldsymbol{n}$, the CP violating asymmetry when the lepton is polarized, with $\boldsymbol{n} = \pm \boldsymbol{e}_i$, is given by

$$
\begin{aligned}
&A_{CP}^\pi(s, \boldsymbol{n} = \pm\boldsymbol{e}_i) \\
&= \frac{1}{2}\frac{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0\left[1 \pm P_i^\pi\right] - (\bar{\Gamma}^\pi/\mathrm{d}s)_0\left[1 \pm \bar{P}_i^\pi\right]}{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 + (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0} \ ,
\end{aligned}
$$

or, by making use of the replacements in (33) and (37) we further obtain

$$
\begin{aligned}
&A_{CP}^\pi(s, \boldsymbol{n} = \pm\boldsymbol{e}_i) \\
&= \frac{1}{2}\left\{\frac{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0}{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0}\right. \\
&\qquad \left. \pm \frac{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 P_i^\pi - ((\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 P_i^\pi)\,|_{\lambda_u \to \lambda_u^*}}{(\mathrm{d}\Gamma^\pi/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\pi/\mathrm{d}s)_0}\right\} \\
&= \frac{1}{2}\left\{A_{CP}^\pi(s) \pm \delta {A_{CP}^\pi}^i(s)\right\} \ . \qquad (38)
\end{aligned}
$$

The $\delta A_{CP}^i(s)$ terms in (38) describe the modifications to the unpolarized decay width, which can be written as

$$
\delta {A_{CP}^\pi}^i(s) = \frac{-2\mathrm{Im}[\lambda_u]\delta\Sigma_\pi^i(s)}{\Delta_\pi(s) + 2\mathrm{Im}[\lambda_u]\Sigma_\pi(s)} \ , \quad (39)
$$

where

$$
\delta\Sigma_\pi^{\mathrm{L}}(s) = \frac{2}{3}\ m_B^2\ v\ \lambda_\pi\ {f^+}^2 \mathrm{Im}[\xi_2 C_{10}^*] \, , \quad (40)
$$

$$
\begin{aligned}
\delta\Sigma_\pi^{\mathrm{T}}(s) = &\frac{m_B^2\ t\ \pi\ \sqrt{\lambda_\pi}}{2\sqrt{s}} \qquad (41) \\
&\times\left((1-r_\pi){f^+}^2 + s f^+ f^-\right)\mathrm{Im}[\xi_2 C_{10}^*] \, ,
\end{aligned}
$$

$$
\delta\Sigma_\pi^{\mathrm{N}}(s) = 0 \, . \quad (42)
$$

3.2 CP violating asymmetry in $B \to \rho\ell^+\ell^-$ decay

In $B \to \rho\ell^+\ell^-$ decay with unpolarized final leptons, CP violating differential decay width asymmetry is defined as in (32) with the replacement $\Delta_\pi \to \Delta_\rho$ and $\mathrm{d}\Gamma^\pi/\mathrm{d}s \to \mathrm{d}\Gamma^\rho/\mathrm{d}s$. Using (29), (32) and (33), the CP violating asymmetry is given as

$$A^\rho_{CP}(s) = \frac{-\mathrm{Im}[\lambda_u]\Sigma_\rho(s)}{2\Delta_\rho + \mathrm{Im}[\lambda_u]\Sigma_\rho(s)} \ , \tag{43}$$

where

$$\begin{aligned}\Sigma_\rho(s) = {} & \frac{4(s+2t)}{3r_\rho s(1+r_\rho)} \\ & \times\big\{m_{B\rho}\mathrm{Im}[\xi_2] \\ & \times\left[A_1 m_{B\rho}^2\left(c_2 m_B^2(-1+r_\rho+s)\lambda_\rho + c_1(12r_\rho s+\lambda_\rho)\right)\right. \\ & +m_B^2\lambda_\rho \\ & \left.\times\left(8m_\rho^2 r_\rho s V c_3 + A_2(c_1(-1+r_\rho+s)+c_2 m_B^2\lambda_\rho)\right)\right] \\ & -2\mathrm{Im}[\xi_1\xi_2^*] \\ & \times\left[2A_1A_2 m_B^2 m_{B\rho}^2\lambda_\rho(-1+r_\rho+s)\right. \\ & \quad +A_1^2 m_{B\rho}^4(12r_\rho s+\lambda_\rho) \\ & \quad \left.+m_B^4\lambda_\rho(8r_\rho sV^2+A_2^2\lambda_\rho)\right]\big\} \ ,\end{aligned} \tag{44}$$

with $m_{B\rho} \equiv m_B + m_\rho$ and

$$\begin{aligned} c_1 &= \frac{8m_b C_7^{\mathrm{eff}}}{q^2}(m_B^2 - m_\rho^2)T_2 \ , \\ c_2 &= 8\frac{m_b}{q^2}C_7^{\mathrm{eff}}\left(T_2 + \frac{q^2}{m_B^2-m_\rho^2}T_3\right), \\ c_3 &= \frac{8m_b C_7^{\mathrm{eff}}}{q^2}T_1 \ . \end{aligned} \tag{45}$$

When one of the leptons is polarized in $B \to \rho\ell^+\ell^-$ decay, the CP violating asymmetry can be defined as follows:

$$A^\rho_{CP}(s,\boldsymbol{n}) = \frac{\mathrm{d}\Gamma^\rho(s,\boldsymbol{n})/\mathrm{d}s - \mathrm{d}\bar{\Gamma}^\rho(s,\bar{\boldsymbol{n}}=-\boldsymbol{n})/\mathrm{d}s}{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 + (\mathrm{d}\bar{\Gamma}^\rho/\mathrm{d}s)_0} \ , \tag{46}$$

where

$$\begin{aligned}\frac{\mathrm{d}\Gamma^\rho(s,\boldsymbol{n})}{\mathrm{d}s} &= \frac{\mathrm{d}\Gamma(B\to\rho\ell^+\ell^-(\boldsymbol{n}))}{\mathrm{d}s} \ , \\ \frac{\mathrm{d}\bar{\Gamma}^\rho(s,\bar{\boldsymbol{n}})}{\mathrm{d}s} &= \frac{B\to\rho\ell^+(\bar{\boldsymbol{n}})\ell^-)}{\mathrm{d}s} \ .\end{aligned}$$

Here, $\bar{\boldsymbol{n}}$ is the spin direction of the ℓ^+ in the $\bar{B} \to \bar{\rho}\ell^+\ell^-$ decay. From the expression for the polarized differential decay width in the $B \to \rho\ell^+\ell^-$ decay given by (30), the width for the corresponding CP conjugated process reads

$$\frac{\mathrm{d}\bar{\Gamma}^\rho(s,\bar{\boldsymbol{n}})}{\mathrm{d}s} = \frac{1}{2}\left(\frac{\mathrm{d}\bar{\Gamma}^\rho}{\mathrm{d}s}\right)_0\left[1+\bar{P}_i^\rho\bar{\boldsymbol{e}}_i\cdot\bar{\boldsymbol{n}}\right] \ . \tag{47}$$

Inserting (30) and (47) into (46), and setting $\bar{\boldsymbol{n}} = \boldsymbol{n}$, the CP violating asymmetry when the lepton is polarized, with $\boldsymbol{n} = \pm\boldsymbol{e}_i$, is given by

$$\begin{aligned}& A^\rho_{CP}(s,\boldsymbol{n}=\pm\boldsymbol{e}_i) \\ & = \frac{1}{2}\frac{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0\left[1\pm P_i^\rho\right] - (\bar{\Gamma}^\rho/\mathrm{d}s)_0\left[1\pm\bar{P}_i^\rho\right]}{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 + (\mathrm{d}\bar{\Gamma}^\rho/\mathrm{d}s)_0} \ ,\end{aligned}$$

or, by making use of the replacements in (33) and (37) with $\pi \to \rho$ we further obtain

$$\begin{aligned}& A^\rho_{CP}(s,\boldsymbol{n}=\pm\boldsymbol{e}_i) \\ & = \frac{1}{2}\left\{\frac{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\rho/\mathrm{d}s)_0}{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\rho/\mathrm{d}s)_0}\right. \\ & \qquad \left.\pm\frac{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 P_i^\rho - ((\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 P_i^\rho)\mid_{\lambda_u\to\lambda_u^*}}{(\mathrm{d}\Gamma^\rho/\mathrm{d}s)_0 - (\mathrm{d}\bar{\Gamma}^\rho/\mathrm{d}s)_0}\right\} \\ & = \frac{1}{2}\left\{A^\rho_{CP}(s) \pm \delta A^{\rho\ i}_{CP}(s)\right\} \ .\end{aligned} \tag{48}$$

The $\delta A^{\rho\ i}_{CP}(s)$ terms in (48) describe the modifications to the unpolarized decay width, which can be written as

$$\delta A^{\rho\ i}_{CP}(s) = \frac{\mathrm{Im}\,\lambda_u\ \delta\Sigma_\rho^i(s)}{\Delta_\rho(s)+\bar{\Delta}_\rho(s)} \ , \tag{49}$$

where

$$\begin{aligned}\delta\Sigma_\rho^{\mathrm{L}}(s) = {} & \frac{4m_B v}{3r_\rho(1+\sqrt{r_\rho})} \\ & \times\mathrm{Im}[\xi_2]\left\{A_1 m_{B\rho}^2\right. \\ & \times\left(c_2' m_B^2(-1+r+s)\lambda_\rho + c_1'(12r_\rho s+\lambda_\rho)\right) \\ & +m_B^2\lambda_\rho \\ & \left.\times\left(8m_\rho^2 r_\rho s V c_4' + A_2(c_1'(-1+r_\rho+s)+c_2' m_B^2\lambda_\rho)\right)\right\} \ ,\end{aligned} \tag{50}$$

$$\begin{aligned}\delta\Sigma_\rho^{\mathrm{T}}(s) = {} & \frac{m_B^2 m_\ell \pi}{r_\rho(1+\sqrt{r_\rho})}\frac{\sqrt{\lambda_\rho}}{\sqrt{s}} \\ & \times\left\{-A_1 m_{B\rho}^2\mathrm{Im}[\xi_2]\left[(-1+r_\rho+s)c_1'/m_B^2\right.\right. \\ & \quad +(-1+r_\rho+s)(-1+r_\rho)c_2' \\ & \quad \left.+s(8r_\rho c_3 + (-1+r_\rho+s)c_3')\right] \\ & +\mathrm{Im}[\xi_2] \\ & \times\left[8r_\rho sVc_1 - A_2\lambda_\rho\left(c_1' + m_B^2((r_\rho-1)c_2' - sc_3')\right)\right] \\ & \left.-32m_{B\rho}r_\rho sA_1V\mathrm{Im}[\xi_1\xi_2^*]\right\} \ ,\end{aligned} \tag{51}$$

$$\begin{aligned}\delta\Sigma_\rho^{\mathrm{N}}(s) = {} & \frac{m_B^2 m_\ell \pi v}{2r_\rho(1+\sqrt{r_\rho})}\sqrt{\lambda_\rho}\sqrt{s}\mathrm{Re}[\xi_2] \\ & \times\left\{(-A_2D_1 + A_1D_2m_{B\rho}^2)(-1-3r_\rho+s)\right. \\ & \quad -m_{B\rho}(-1+r_\rho+s)(A_1D_3m_{B\rho} - 2D_1T_3/m_b) \\ & \quad \left.-8r_\rho(A_1c_4'm_{B\rho}^2 + c_1'V)\right\} \ ,\end{aligned} \tag{52}$$

where

$$c_1' = -2m_{B\rho} A_1 C_{10} \quad , \quad c_2' = -2A_2 C_{10}/m_{B\rho} \, ,$$
$$c_3' = -4T_3 C_{10}/m_b \quad , \quad c_4' = -2V C_{10}/m_{B\rho} \, , \tag{53}$$

and

$$D_1 = F(C_9^{\text{eff}} \to 0) \; , \; D_2 = G(C_9^{\text{eff}} \to 0) \; ,$$
$$D_3 = H(C_9^{\text{eff}} \to 0) \; . \tag{54}$$

4 Numerical results and discussion

In this section we present the numerical analysis of both the exclusive decays $B \to \pi \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ for $\ell = e, \tau$. We do not present the results for $\ell = \mu$ because they are similar to the ones for $\ell = e$. The input parameters we used in our numerical analysis are as follows:

$$\begin{aligned} & m_B = 5.28\,\text{GeV}\, , \; m_b = 4.8\,\text{GeV}\, , \; m_c = 1.4\,\text{GeV}\, , \\ & m_\tau = 1.78\,\text{GeV}\, , \; m_e = 0.511\,\text{MeV}, \; m_\mu = 0.106\,\text{GeV}, \\ & m_\pi = 0.14\,\text{GeV}\, , \; m_\rho = 0.77\,\text{GeV}\, , \\ & m_d = m_u = m_\pi = 0.14\,\text{GeV}\, , \; |V_{cb}| = 0.044\, , \\ & \alpha^{-1} = 129\, , \; G_\text{F} = 1.17 \times 10^{-5}\,\text{GeV}^{-2}\, , \\ & \tau_B = 1.56 \times 10^{-12}\,\text{s}\, . \end{aligned} \tag{55}$$

Using the Wolfenstein parametrization of the CKM matrix [40], λ_u in (6) can be written as

$$\lambda_u = \frac{\rho(1-\rho) - \eta^2 - \mathrm{i}\eta}{(1-\rho)^2 + \eta^2} + O(\lambda^2). \tag{56}$$

Furthermore, we use the relation

$$\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2} = \lambda^2[(1-\rho)^2 + \eta^2] + \mathcal{O}(\lambda^4), \tag{57}$$

where $\lambda = \sin\theta_\text{C} \simeq 0.221$ and adopt the values of the Wolfenstein parameters as $\rho = 0.25$ and $\eta = 0.34$.

In order to obtain numerical results for the $B \to \pi \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ decays, we also need the numerical values of the decay form factors. The literature on this subject is very rich; we give some references here. For $B \to \pi(\rho)$ the form factors are calculated in the constituent quark model [41] and using the light-cone QCD sum rules [42,43] ([44,45]). In [46] the results of the lattice QCD calculations are given for the $B \to \pi, \rho$ form factors, while the perturbative QCD approach [47] and the so-called large-energy effective theory [48] have also been employed to calculate these form factors.

4.1 Numerical results of the exclusive $B \to \pi \ell^+ \ell^-$ decay

In order to obtain numerical results for the $B \to \pi \ell^+ \ell^-$ decay, we have made use of the results of the constituent quark model [41], where the form factors f_T and f_+ can be parameterized as

$$f(q^2) = \frac{f(0)}{(1 - q^2/M^2)[1 - \sigma_1\, q^2/M^2 + \sigma_2\, q^4/M^4]}\, . \tag{58}$$

In this model, f_- is redefined as

$$F_0 = f_+ + \frac{q^2}{(p_B + p_\pi)q} f_- \, , \tag{59}$$

and its interpolation formula is given as

$$f(q^2) = \frac{f(0)}{[1 - \sigma_1\, q^2/M^2 + \sigma_2\, q^4/M^4]}\, . \tag{60}$$

The parameters $f(0)$, σ_1 and σ_2 can be found in Table 1. Note that for f_+ and f_T a simple monopole two parameter formula is used, viz. $\sigma_2 = 0$.

In Fig. 1 we present our results of the differential branching ratios (dBR/ds) of the unpolarized and longitudinally polarized $B \to \pi e^+ e^-$ decay. dBR/ds for the $\boldsymbol{n} = -\boldsymbol{e}_\text{L}$ polarized case is close to the one of unpolarized decay, which implies that the decay is naturally left handed. dBR/ds for the $\boldsymbol{n} = +\boldsymbol{e}_\text{L}$ polarization case is far below dBR/ds for the unpolarized one. Thus, $\boldsymbol{n} = +\boldsymbol{e}_\text{L}$ polarized $B \to \pi e^+ e^-$ decay corresponds to a wrong sign decay.

In Figs. 2 and 3, we plot the longitudinally polarized asymmetries and the unpolarized CP violating asymmetry together with $-\delta A_{CP}^\text{L}$ of the $B \to \pi e^+ e^-$ decay, respectively. From Fig. 2 it can be observed that $A_{CP}(\boldsymbol{n} = -\boldsymbol{e}_\text{L})$ is much larger than $A_{CP}(\boldsymbol{n} = +\boldsymbol{e}_\text{L})$. It is also observed from Fig. 3 that $-\delta A_{CP}^\text{L}$ exceeds the unpolarized A_{CP} in some kinematical regions but is mostly comparable with it. Particularly, in the region $(2m_\ell/m_B)^2 \le s \le ((m_{J/\psi} -$

Table 1. $B \to \pi$ transition form factors in the constituent quark model

	$f(0)$	σ_1	σ_2
f_+	0.29	0.48	
F_0	0.29	0.76	0.28
f_T	0.28	0.48	

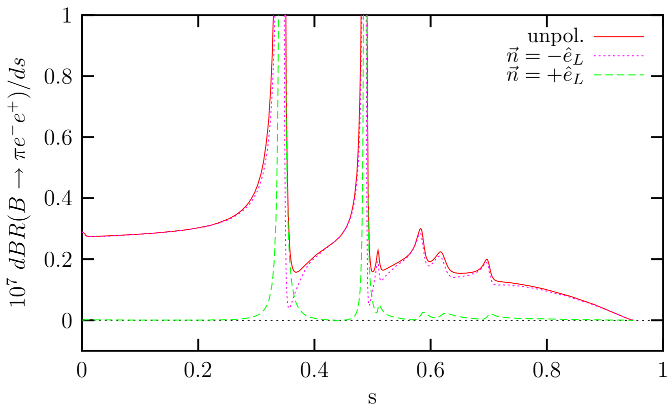

Fig. 1. Polarized and unpolarized differential branching ratios for $B \to \pi e^+ e^-$ decay

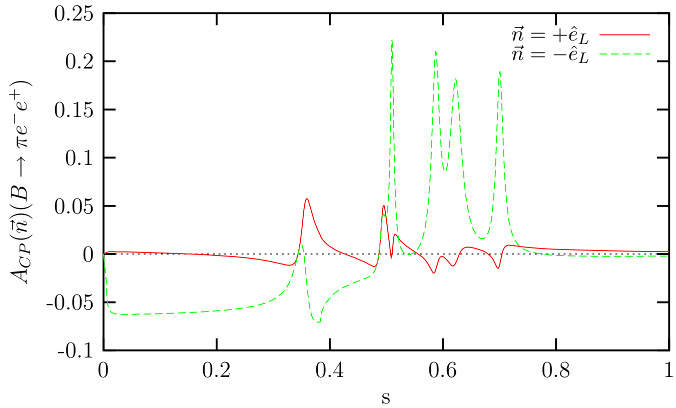

Fig. 2. Longitudinally polarized CP violating asymmetries for $B \to \pi e^+ e^-$ decay

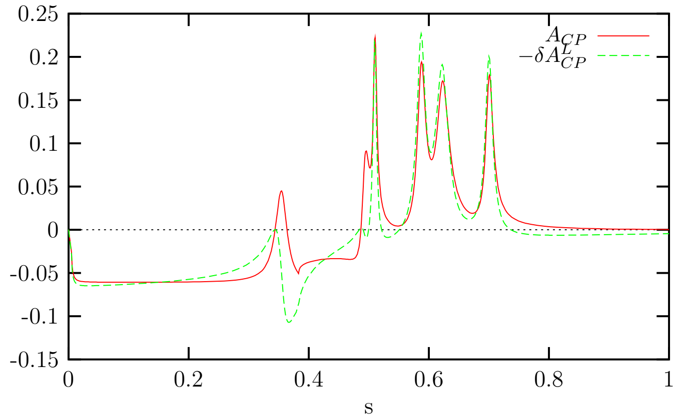

Fig. 3. Unpolarized CP violating asymmetry and longitudinally polarized quantity $-\delta A^{\mathrm{L}}_{CP}$ for $B \to \pi e^+ e^-$ decay

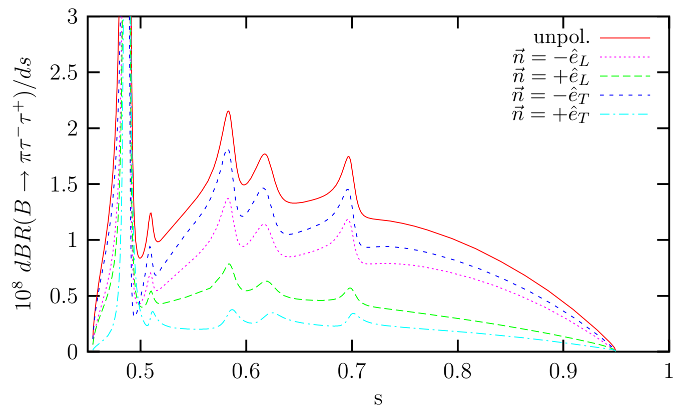

Fig. 4. Polarized and unpolarized differential branching ratios for $B \to \pi\tau^+\tau^-$ decay

$0.02)/mB)^2$, which is free of any resonance contribution, we find that $\delta A^{\mathrm{L}}_{CP}$ and A_{CP} are about 6%. We see also from Fig. 3 that in the resonance region $\delta A^{\mathrm{L}}_{CP}$ can reach values up to 25%.

In Fig. 4, we present $\mathrm{dBR}/\mathrm{d}s$ for the decay $B \to \pi\tau^+\tau^-$ for unpolarized, longitudinally and transversely polarized τ leptons. We observe that $\mathrm{dBR}/\mathrm{d}s$ for $\boldsymbol{n} = -\boldsymbol{e}_{\mathrm{L}}$ and $\boldsymbol{n} = -\boldsymbol{e}_{\mathrm{T}}$ are close to unpolarized $\mathrm{dBR}/\mathrm{d}s$, while it becomes smaller for $\boldsymbol{n} = +\boldsymbol{e}_{\mathrm{L}}$ and $\boldsymbol{n} = +\boldsymbol{e}_{\mathrm{T}}$. The $\boldsymbol{n} = +\boldsymbol{e}_{\mathrm{T}}$ polarization case gives a very small $\mathrm{dBR}/\mathrm{d}s$ as compared to the unpolarized decay and thus can be identified as wrong sign decay.

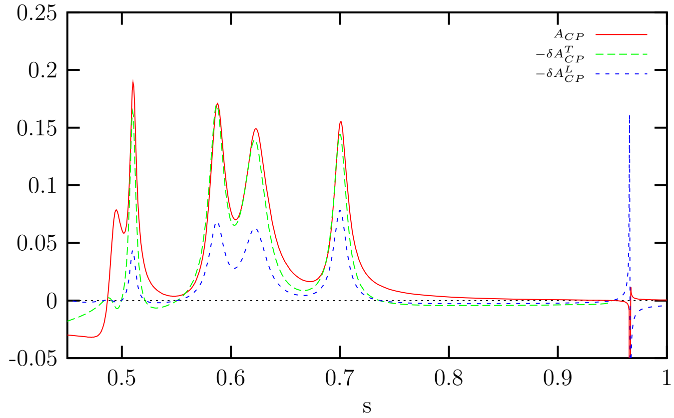

Fig. 5. Unpolarized A_{CP} and $-\delta A^i_{CP}$ with $i = \mathrm{L}, \mathrm{T}$ for $B \to \pi\tau^+\tau^-$ decay

In Fig. 5, we plot the unpolarized A_{CP} and longitudinally and transversely polarized $-\delta A_{CP}$ of the decay $B \to \pi\tau^+\tau^-$. We observe that although $\delta A^{\mathrm{L}}_{CP}$ is small, $\delta A^{\mathrm{T}}_{CP}$ is very close to A_{CP} especially in the resonance regions. Therefore, we can conclude that $A^{\mathrm{L}}_{CP}(\boldsymbol{n} = +\boldsymbol{e}_i) \simeq A^{\mathrm{L}}_{CP}(\boldsymbol{n} = -\boldsymbol{e}_i)$. The asymmetries reach a maximum value of 13%.

4.2 Numerical results of the exclusive $B \to \rho\ell^+\ell^-$ decay

In our numerical calculation for $B \to \rho\ell^+\ell^-$ decay, we use a three parameter fit of the light-cone QCD sum rule [44] which can be written in the following form:

$$F(q^2) = \frac{F(0)}{1 - a_F\, q^2/m_B^2 + b_F (q^2/m_B^2)^2}\,, \tag{61}$$

where the values of the parameters $F(0)$, a_F and b_F are given in Table 2. The form factors A_0 and A_3 can be found from the following parametrization:

$$\begin{aligned} A_0 &= A_3 - \frac{T_3\, q^2}{m_\rho m_b}, \\ A_3 &= \frac{m_B + m_\rho}{2m_\rho} A_1 - \frac{m_B - m_\rho}{2m_\rho} A_2. \end{aligned} \tag{62}$$

In Fig. 6 we present $\mathrm{dBR}/\mathrm{d}s$ for the decay $B \to \rho e^+ e^-$ with unpolarized and longitudinally polarized electrons. It can be seen from this figure that the polarized spectrum for $\boldsymbol{n} = -\boldsymbol{e}_{\mathrm{L}}$ almost coincides with unpolarized spectrum, whereas the polarized $\boldsymbol{n} = +\boldsymbol{e}_{\mathrm{L}}$ spectrum is far below the unpolarized one. So, decay is naturally left handed in the SM.

In Figs. 7 and 8 we plot the longitudinally polarized CP violating asymmetries, $A_{CP}(\boldsymbol{n})$ with $\boldsymbol{n} = -\boldsymbol{e}_{\mathrm{L}}$ and $\boldsymbol{n} = +\boldsymbol{e}_{\mathrm{L}}$, and unpolarized A_{CP} together with the polarized quantity $\delta A^{\mathrm{L}}_{CP}$ for the decay $B \to \rho e^+ e^-$, respectively.

Table 2. $B \to \rho$ transition form factors in a three parameter fit

	$F(0)$	a_F	b_F
$A_1^{B\to\rho}$	0.26 ± 0.04	0.29	-0.415
$A_2^{B\to\rho}$	0.22 ± 0.03	0.93	-0.092
$V^{B\to\rho}$	0.34 ± 0.05	1.37	0.315
$T_1^{B\to\rho}$	0.15 ± 0.02	1.41	0.361
$T_2^{B\to\rho}$	0.15 ± 0.02	0.28	-0.500
$T_3^{B\to\rho}$	0.10 ± 0.02	1.06	-0.076

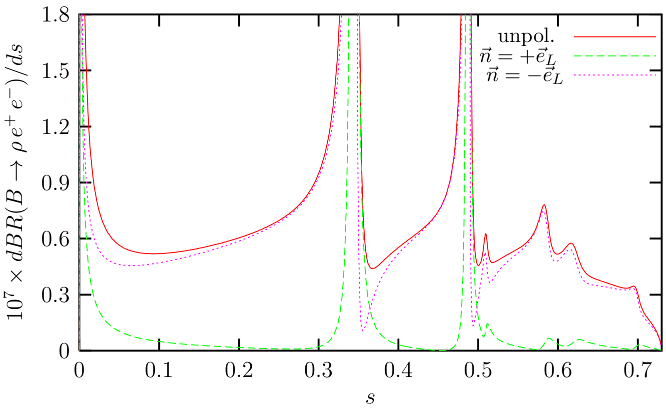

Fig. 6. Polarized and unpolarized differential branching ratios for $B \to \rho e^+e^-$ decay

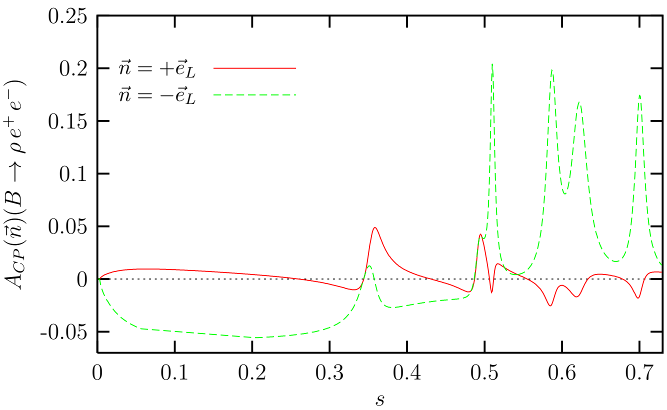

Fig. 7. Longitudinally polarized CP violating asymmetries for $B \to \rho e^+e^-$ decay

As can be seen from Fig. 7, $A_{CP}(\boldsymbol{n} = -\boldsymbol{e}_\mathrm{L})$ is much larger than $A_{CP}(\boldsymbol{n} = +\boldsymbol{e}_\mathrm{L})$. We see from Fig. 8 that polarized CP violating asymmetry δA_{CP}^L becomes larger than its unpolarized counterpart in some kinematic regions. Particularly, in the region $(2m_\ell/m_B)^2 \leq s \leq ((m_{J/\psi} - 0.02)/mB)^2$, which is free of resonance contributions, we find that δA_{CP}^L is about 6%, while the unpolarized A_{CP} is about 4%. We see also from Fig. 8 that in the resonance region δA_{CP}^L can reach values up to 25%.

In Fig. 9, we present the dBR/ds for the decay $B \to \rho\tau^+\tau^-$ for unpolarized, longitudinally, transversely and normally polarized τ leptons. We see that dBR/ds for $\boldsymbol{n} = +\boldsymbol{e}_\mathrm{N}$ and $\boldsymbol{n} = -\boldsymbol{e}_\mathrm{N}$ almost coincide, while for $\boldsymbol{n} = \pm\boldsymbol{e}_\mathrm{L}$, the state with $\boldsymbol{n} = -\boldsymbol{e}_\mathrm{L}$ is much more comparable with the unpolarized dBR/ds with respect to the one with $\boldsymbol{n} = +\boldsymbol{e}_\mathrm{L}$.

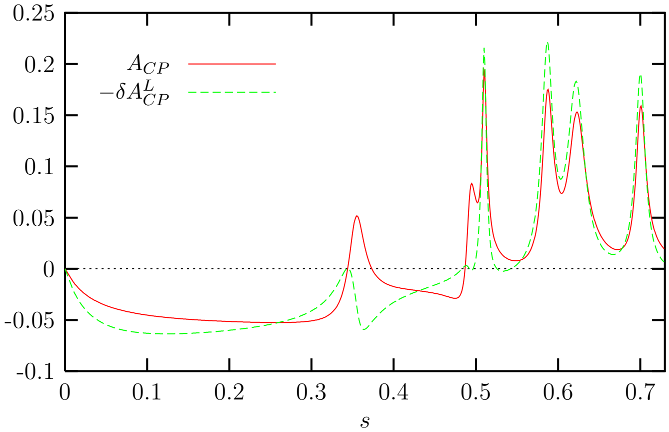

Fig. 8. Unpolarized CP violating asymmetry and longitudinally polarized quantity $-\delta A_{CP}^\mathrm{L}$ for $B \to \rho e^+e^-$ decay

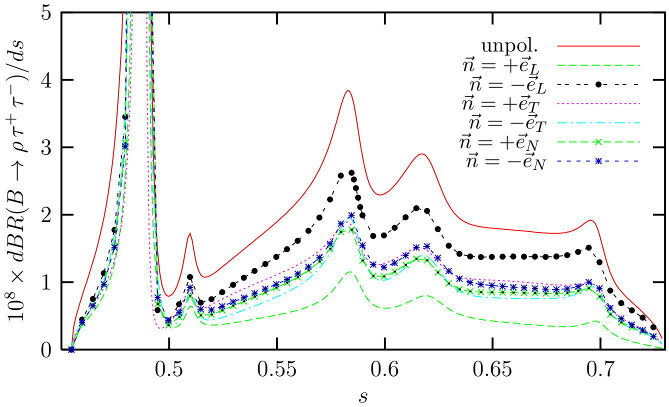

Fig. 9. Polarized and unpolarized differential branching ratios for $B \to \rho\tau^+\tau^-$ decay

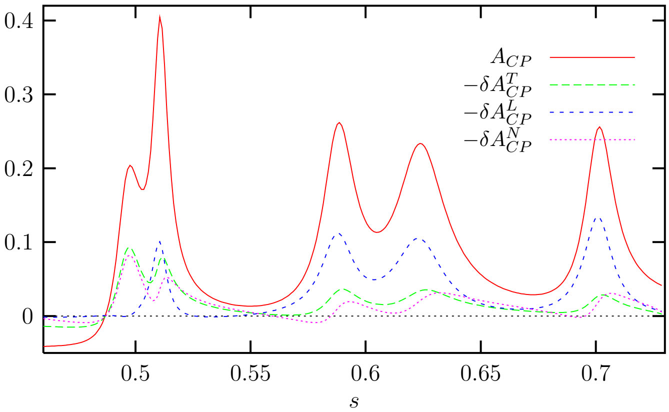

Fig. 10. Unpolarized A_{CP} and $-\delta A_{CP}^i$ with $i = \mathrm{L, T, N}$ for $B \to \rho\tau^+\tau^-$ decay

In Fig. 10, we give longitudinally, transversely and normally polarized and unpolarized CP violating rate asymmetries for the decay $B \to \rho\tau^+\tau^-$. We observe that δA_{CP}^T and δA_{CP}^N are both smaller than δA_{CP}^L. Therefore, we can conclude that $A_{CP}(\boldsymbol{n} = +\boldsymbol{e}_i) \simeq A_{CP}(\boldsymbol{n} = -\boldsymbol{e}_i)$ for $i = \mathrm{T, N}$, while for $i = \mathrm{L}$ $A_{CP}(\boldsymbol{n} = +\boldsymbol{e}_\mathrm{L})$ is quite small as compared to its counterpart with $\boldsymbol{n} = -\boldsymbol{e}_\mathrm{L}$.

5 Conclusion

We have calculated the polarized decay rate and CP violating asymmetries of the decays $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$. For $\ell = e$, which is in the specific polarized channel $\boldsymbol{n} = -\boldsymbol{e}_{\rm L}$, the decay rate is comparable to the one of the unpolarized decay. The normal and the transverse polarizations are proportional to the mass of the lepton and therefore can be significant for the τ lepton only. For the $B \to \pi\tau^+\tau^-$ decay, $\boldsymbol{n} = \pm\boldsymbol{e}_{\rm L}$ and for the $B \to \rho\tau^+\tau^-$ decay $\boldsymbol{n} = \pm\boldsymbol{e}_{\rm T}$ and $\boldsymbol{n} = \pm\boldsymbol{e}_{\rm N}$ give similar widths. For the rest, which are defined as the wrong sign decays, the decay rates and the CP violating asymmetries are much lower as compared to the unpolarized ones.

In conclusion, we studied the decay rate and the CP violating asymmetry of the exclusive $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ decays in the case where one of the final leptons is polarized. Since the SM is naturally left handed, the wrong sign decays, in particular $\boldsymbol{n} = +\boldsymbol{e}_{\rm L}$ polarized $B \to (\pi, \rho)e^+e^-$, $\boldsymbol{n} = +\boldsymbol{e}_{\rm T}$ polarized $B \to \pi\tau^+\tau^-$ and $\boldsymbol{n} = +\boldsymbol{e}_{\rm L}$ polarized $B \to \rho\tau^+\tau^-$ decays, are more sensitive to new physics. Taking into account the typical branching ratios and CP violating asymmetries, 10^{10}–10^{11} $B\bar{B}$ pairs are needed for the observation of CP violation in the exclusive channels [9], which is a challenging task for the future hadron colliders. An unexpected large asymmetry in these channels and the wrong sign decays would be very significant in the search for new physics beyond the SM.

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References

1. J.L. Hewett, in Proceedings of the 21st Annual SLAC Summer Institute, SLAC-PUB-6521, 1994, edited by L. De Porcel, C. Dunwoode; J.L. Hewett, hep-ph/9803370
2. J.P. Alexander et al. [CLEO Collaboration], Phys. Rev. Lett. **77**, 5000 (1996); Phys. Rev. D **61**, 052001 (2000)
3. J. Kaneko et al. [Belle Collaboration], Phys. Rev. Lett. **90**, 021801 (2003)
4. A. Ali, E. Lunghi, C. Greub, G. Hiller, Phys. Rev. D **66**, 034002 (2002)
5. B. Aubert et al. [BABAR Collaboration], hep-ex/0308016
6. T.M. Aliev, D.A. Demir, E. Iltan, N.K. Pak, Phys. Rev. D **54**, 851 (1996)
7. D.S. Du, M.Z. Yang, Phys. Rev. D **54**, 882 (1996)
8. F. Krüger, L.M. Sehgal, Phys. Rev. D **55**, 2799 (1997)
9. F. Krüger, L.M. Sehgal, Phys. Rev. D **56**, 5452 (1997); D **60**, 099905 (1999) (E)
10. G. Erkol, G. Turan, J. Phys. G **28**, 2983 (2002)
11. G. Erkol, G. Turan, Eur. Phys. J. C **28**, 243 (2003)
12. T.M. Aliev, M. Savcı, Phys. Rev. D **60**, 014005 (1999)
13. E.O. Iltan, Int. J. Mod. Phys. A **14**, 4365 (1999)
14. G. Erkol, G. Turan, J. High Energy Phys. **02**, 015 (2002)
15. S. Rai Choudhury, N. Gaur, Phys. Rev. D **66**, 094015 (2002)
16. S. Rai Choudhury, N. Gaur, hep-ph/0207353
17. K.S. Babu, K.R.S. Balaji, I. Schienbein, Phys. Rev. D **68**, 014021 (2003)
18. T.M. Aliev, V. Bashiry, M. Savci, Eur. Phys. J. C **31**, 511 (2003)
19. B. Grinstein, R. Springer, M. Wise, Nucl. Phys. B **339**, 269 (1990)
20. A.J. Buras, M. Misiak, M. Münz, S. Pokorski, Nucl. Phys. B **424**, 372 (1994)
21. M. Misiak, Nucl. Phys. B **393**, 23 (1993); B **439**, 461 (1993) (E); A.J. Buras, M. Münz, Phys. Rev. D **52**, 186 (1995)
22. G. Buchalla, A. Buras, M. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996)
23. K. Adel, Y.P. Yao, Phys. Rev. D **49**, 4945 (1994)
24. C. Greub, T. Hurth, Phys. Rev. D **56**, 2934 (1997)
25. A.J. Buras, A. Kwiatkowski, N. Pott, Nucl. Phys. B **517**, 353 (1998)
26. C. Bobeth, M. Misiak, J. Urban, Nucl. Phys. B **574**, 291 (2000)
27. P. Gambino, M. Gorbahn, U. Haisch, Nucl. Phys. B **673**, 238 (2003)
28. H.H. Asatryan, H.M. Asatrian, C. Greub, M. Walker, Phys. Rev. D **65**, 074004 (2002)
29. H.H. Asatryan, H.M. Asatrian, C. Greub, M. Walker, Phys. Rev. D **66**, 034009 (2002)
30. A. Ghinculov, T. Hurth, G. Isidori, Y.P. Yao, Eur. Phys. J. C **33**, S288 (2004)
31. A. Ghinculov, T. Hurth, G. Isidori, Y.P. Yao, Nucl. Phys. B **648**, 254 (2003)
32. H.M. Asatrian, K. Bieri, C. Greub, A. Hovhannisyan, Phys. Rev. D **66**, 094013 (2002)
33. A. Ghinculov, T. Hurth, G. Isidori, Y.P. Yao, Nucl. Phys. Proc. Suppl. **116**, 284 (2003)
34. H.M. Asatrian, H.H. Asatryan, A. Hovhannisyan, V. Poghosyan, Mod. Phys. Lett. A **19**, 603 (2004)
35. T. Hurth, Rev. Mod. Phys. **75**, 1159 (2003)
36. H.M. Asatrian, K. Bieri, C. Greub, M. Walker, Phys. Rev. D **69**, 074007 (2004)
37. A. Ali, T. Mannel, T. Morozumi, Phys. Lett. B **273**, 505 (1991)
38. C.Q. Geng, C.P. Kao, Phys. Rev. D **54**, 5636 (1996)
39. T.M. Aliev, M.K. Çakmak, M. Savcı, Nucl. Phys. B **607**, 305 (2001)
40. L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983)
41. D. Melikhov, B. Stech, Phys. Rev. D **62**, 014006 (2000)
42. P. Ball, JHEP **9809**, 005 (1998); **9901**, 010 (1999)
43. P. Ball, hep-ph/0308249
44. P. Ball, V.M. Braun, Phys. Rev. D **58**, 094016 (1998)
45. P. Ball, hep-ph/0412079
46. L.D. Debbio et al. [UKQCD Collaboration], Phys. Lett. B **416**, 392 (1998)
47. C.D. Lü, M.Z. Yang, Eur. Phys. J. C **28**, 515 (2003)
48. M. Beneke, T. Feldmann, Nucl. Phys. B **592**, 3 (2001)